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MIMIMUM REQUIRED SIGNAL FOR LIMARS EXPERIMENTS

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FOREWORD

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ABSTRACT

This report determines the minimum detectable signal required to perform the LIMARS experiment. LIMARS is a direct detection laser radar using a novel receiver scheme to produce a pixel-registered range image without scanning. This work was motivated by the need to size the transmitter laser which, for this experimental system, is a Nd:YAG laser operating at $1.06\mu m$. The laser must produce at least the minimum detectable power at the detector for the experimental conditions. The minimum detectable power is that power which just exceeds the noise for given probabilities of detection and false alarm. I included contributions of noise from scene emission, scene reflection, and internal sources, and I considered experiments in the Bldg 620 Tower and the Bldg 622 Collimator. For the Tower experiment, in the worst case of reflected sunlight there are 410 rms noise electrons per pixel. In the Collimator experiment, internal noise of 100 rms noise electrons per pixel dominates.

I wrote a computer program to plot the probability of detection as a function of signal electrons for given probability of false alarm and number of noise electrons using Poisson statistics. Speckle was not included in this program. I verified the output by comparison with other published results and with a Gaussian approximation. I used the program to calculate the number of signal electrons needed in the two experimental cases for 90% probability of detection and 0.1% probability of false alarm. The Tower experiment requires 1800 received signal electrons per pixel, or 1.7×10^{-14}J per pixel at $1.06\mu m$ with a 2% detector quantum efficiency. The Collimator experiment requires 440 received signal electrons per pixel, or 4.1×10^{-15}J per pixel. These values for minimum detectable energy at the detector can then be used in the laser radar range equation to determine the output required from the transmitter laser.

BASIC LASER RADAR EQUATION

$$P_R = \frac{P_T}{R^2\Omega_T} \frac{\rho A_{Targ}}{R^2\Omega_{Targ}} A_R \quad T_{RT}$$

where

P_R	= Received Power	ρ	= Target Reflectivity
A_R	= Receiver Effective Area	A_{Targ}	= Target Effective Area
P_T	= Transmitted Power	Ω_{Targ}	= Target Solid Angle
Ω_T	= Transmitter Solid Angle	T_{RT}	= Round Trip Loss
R	= Range		

Agrees with radar formulation of the range equation with the definition of Antenna Gain G_t and Target Cross Section σ given by:

$$\frac{G_t}{4\pi} = \frac{1}{\Omega_T} \qquad\qquad \frac{\sigma}{4\pi} = \frac{\rho A_{Targ}}{\Omega_{Targ}}$$

- Transmitter Power Required:

$$P_T \geq \frac{P_{R,\min} R^4 \Omega_T \Omega_{Targ}}{\rho A_{Targ} A_R T_{RT}}$$

where $P_{R,\min}$ is the Minimum Detectable Power:

$$P_{R,\min} = (NEP)(SNR)$$

where NEP=Noise Equivalent Power and SNR=Signal-to-Noise Ratio required for given probabilities of detection and false alarm.

NOISE POWER

- Noise Power $\propto$ Mean Squared Current Fluctuation $\overline{i_N^2}$:

$$\overline{i_N^2} = \overline{i_S^2} + \overline{i_{BG}^2} + \overline{i_{IN}^2}$$

where $\overline{i_S^2}$ = shot noise due to the signal
$\overline{i_{BG}^2}$ = shot noise due to the background
$\overline{i_{IN}^2}$ = noise due to all internal sources

Since $h\nu \ll kT$ at $\lambda = 1\mu m$, noise due to fluctuation of background power can be ignored.

- **Shot Noise**

$$\overline{i_{SN}^2} = 2q\bar{i}B$$

where q = charge of electron
$\bar{i}$ = average photogenerated current
B = electrical bandwidth of the circuit

The average photogenerated current is related to the power in the incident optical beam by:

$$\bar{i} = \frac{P\eta q}{h\nu}$$

where P = incident optical power
η = detector quantum efficiency
ν = frequency of incident optical radiation

NUMBER OF NOISE ELECTRONS

- A CCD collects, stores, and transfers electrons which are converted to voltage for output. It is common to describe signals in CCDs in terms of number of electrons, N.

- The Number of Electrons N and the Current i are Random Variables related by:

  $$N = \frac{i\tau}{q} \qquad \text{where } \tau = \text{observation time}$$

  The Mean and Variance are related by:

  $$\overline{N} = \frac{\overline{i}\tau}{q} \qquad \mathrm{Var}[N] = \frac{\overline{i^2}\tau^2}{q^2} \qquad \text{where } \overline{i^2} = \text{Variance of } i = \text{rms current fluctuation}$$

  The Equivalent Number of Noise Electrons = Standard Deviation of N = $\sqrt{\mathrm{Var}[N]}$.

- For a Poisson process, the variance equals the mean: $\overline{N} = \mathrm{Var}[N]$.

  So the equivalent number of noise electrons due to shot noise is:

  $$N_{SN} = \sqrt{\overline{N}} = \left(\frac{P\eta\tau}{h\nu}\right)^{\frac{1}{2}}$$

SIGNAL SHOT NOISE

- Average Photogenerated Current due to the Signal:

$$\overline{i_S} = \frac{P_S q \eta}{h \nu_S}$$

- Signal Shot Noise:

$$\overline{i_S^2} = 2q\overline{i_s}B$$

- Since this noise is due to the signal itself, Signal Shot Noise is present even when there is no other noise.

"Quantum Limited Detection" $\Rightarrow$ Signal Shot Noise is only source of noise
"Photon Limited Detection"

BACKGROUND SHOT NOISE

- Average Photogenerated Current due to the Background:

$$\overline{i_{BG}} = \frac{P_{BG} q \eta}{h \nu_S}$$

Valid for narrowband optical filter of bandwidth $\Delta\nu$ centered around ν_S such that $\Delta\nu \ll \nu_S$ and P_{BG} and η are constant over $\Delta\nu$.

- Background Shot Noise:

$$\overline{i_{BG}^2} = 2q\,\overline{i_{BG}}\,B$$

- For imaging, each pixel is well-resolved. Assuming the detector size is matched to the blur circle of the receiver optics, all background power due to the area within a pixel instantaneous field of view is collected by the corresponding detector element.

  Total power received for an individual pixel is:

$$P_{BG} = \eta' \eta_{pol} \int_{A_S} \int_{\Omega_R} L \cos\theta \, d\Omega \, dA$$

where

η'	=	transmission loss between the target and the receiver
η_{pol}	=	loss due to polarization filter
A_S	=	the area covered by the pixel on the object
Ω_R	=	solid angle subtended by the receiver aperture
L	=	object radiance over band of interest [W m^{-2} sr^{-1}]
θ	=	angle between the line of sight and the normal to the plane of the target surface

BACKGROUND SHOT NOISE (CONTINUED)

For a Lambertian object, the Radiance L is constant:

$$L = \frac{W}{\pi}\ \mathrm{W/m^2 sr}$$ where W = object exitance over band of interest [W m^{-2}]

Assuming Lambertian objects and ignoring the obliquity factor $\cos\theta$:

$$P_{BG} = \eta'\eta_{pol}\frac{W}{\pi}A_S\Omega_R = \eta'\eta_{pol}\frac{W}{\pi}A_S\frac{A_R}{R^2}$$

where A_R = receiver aperture area
R = range
and it is assumed that $A_R \ll R^2$

- Noise power per pixel:

$$\overline{i_{BG}^2} = 2q\left(\frac{P_{BG}}{h\nu_S}\eta q\right)B$$

- Number of equivalent noise electrons:

$$N_{BG} = \left(\frac{P_{BG}\eta\tau}{h\nu_S}\right)^{1/2} = \left(\frac{\eta'\eta_{pol}WA_SA_R\eta\tau}{\pi R^2 h\nu_S}\right)^{\frac{1}{2}}$$

SOURCES OF BACKGROUND POWER

Scene Emission:

- Assume objects radiate as Blackbodies at temperature T:

$$W_{Emission} = \int W(\lambda)\mathrm{d}\lambda = \int \frac{2\pi}{\lambda^5} c^2 h \frac{1}{\exp\left(\frac{hc}{\lambda kT}\right) - 1} \mathrm{d}\lambda$$

- Assume narrowband optical filter of bandwidth $\Delta\lambda$ centered around the signal wavelength λ_S such that $\Delta\lambda \ll \lambda_S$:

$$W_{Emission} = \frac{2\pi}{\lambda_S^5} c^2 h \frac{1}{\exp\left(\frac{hc}{\lambda_S kT}\right) - 1} \Delta\lambda$$

- For $\lambda_S = 1.06\mu\mathrm{m}$ and T=300°K:

$$W_{Emission} = 6.2 \times 10^{-6}\,\Delta\lambda \text{ W/m}^2$$

SOURCES OF BACKGROUND POWER (CONTINUED)

Scene Reflection:

- Worst case is Reflected Sunlight. Assume the Sun is a 5770°K Blackbody:

$$W_{Sun} = 2.9 \times 10^{13}\, \Delta\lambda \text{ W/m}^2$$

- Irradiance at the Earth's Surface:

$$W_{Earth} = W_{Sun} \frac{R_{Sun}^2}{R_{E-S}^2} \Gamma = 6.4 \times 10^8\, \Gamma\, \Delta\lambda \text{ W/m}^2$$

where R_{Sun} = radius of the sun = 6.952×10^8m
R_{E-S} = distance from the sun to the Earth = 1.495×10^{11}m
Γ = transmission through the entire atmosphere

Assuming $\Gamma = 0.5$: $W_{Earth} = 3.2 \times 10^8\, \Delta\lambda \text{ W/m}^2$

IR Handbook value:	Radar Handbook value:	Measurement by AARI-3:
$W_{Earth} = 5.3 \times 10^8 \Delta\lambda \text{W/m}^2$	$W_{Earth} = 1 \times 10^9 \Delta\lambda \text{W/m}^2$	$W_{Earth} = 6.1 \times 10^6 \Delta\lambda \text{W/m}^2$

- Emittance due to Scene Reflection:

$$W_{Reflection} = W_{Earth}\rho = 6.4 \times 10^8\, \Gamma\, \rho\, \Delta\lambda \text{ W/m}^2$$

where ρ = scene reflectance (total hemispherical reflectance).

Assuming $\Gamma = 0.5$ and $\rho = 0.5$: $W_{Reflection} = 1.6 \times 10^8\, \Delta\lambda \text{ W/m}^2$

BACKGROUND SHOT NOISE FOR LIMARS EXPERIMENT (TOWER)

$$N_{BG} = \left(\frac{\eta' \eta_{pol} W A_S A_R \eta \tau}{\pi R^2 h \nu_S} \right)^{1/2}$$

Typical values for Tower experiment:

$\eta_{pol} = 0.5$ (1 linear polarization component)
$A_S = 0.225\text{m}^2$ (6" square pixel to resolve 1' cross-range)
$A_R = 0.0314\text{m}^2$ (20cm diameter receiver aperture)
$\eta = 0.02$ (Fairchild CCD222)
$\tau = 30\text{ms}$ (30Hz frame rate)
$R = 2.3\text{km}$ (Bldg 620 Tower to Wright Field Test Range)
$\lambda_S = 1.06\mu\text{m}$

$$N_{BG} = 824.7\sqrt{\eta' W} = 824.7\sqrt{\eta' C \Delta\lambda} \quad \text{where } C = \begin{cases} 1.6 \times 10^{8} & \text{for bright sunlight reflection} \\ 6.2 \times 10^{-6} & \text{for T=300K emission only} \end{cases}$$

$\Delta\lambda$ nm	N_{BG} $(\eta' = 11.6\% - 100\%)$	
	$C = 1.6 \times 10^{8}$	$C = 6.2 \times 10^{-6}$
1	110 - 330	$2 - 6 \times 10^{-5}$
10	360 - 1000	$0.7 - 2 \times 10^{-4}$
100	1100 - 3300	$2 - 6 \times 10^{-4}$

• For Transmitter Sizing, use 10nm filter and sunlit day: $N_{BG} \approx 400$ noise electrons

BACKGROUND SHOT NOISE FOR LIMARS EXPERIMENT (COLLIMATOR)

$$N_{BG} = \left(\frac{\eta' \eta_{pol} W A_S A_R \eta \tau}{\pi R^2 h \nu_S} \right)^{1/2}$$

Typical values for Collimator experiment:

$\eta_{pol} = 0.5$ (1 linear polarization component)
$A_S = 4.22 \times 10^{-6} \mathrm{m}^2$ (2° FOV half-angle / 210,688 pixels)
$A_R = 0.00385 \mathrm{m}^2$ (70mm diameter receiver aperture)
$\eta = 0.02$ (Fairchild CCD222)
$\tau = 30\mathrm{ms}$ (30Hz frame rate)
$R = 15.24\mathrm{m}$ (Length of Bldg 622 Collimator Test Range)
$\lambda_S = 1.06 \mu\mathrm{m}$

$$N_{BG} = 188.7 \sqrt{\eta' W} = 188.7 \sqrt{\eta' C \Delta\lambda} \quad \text{where } C = \begin{cases} 1.6 \times 10^{8} & \text{for bright sunlight reflection} \\ 6.2 \times 10^{-6} & \text{for T=300K emission only} \end{cases}$$

	N_{BG} $(\eta' = 11.6\% - 100\%)$	
$\Delta\lambda$ nm	$C = 1.6 \times 10^8$	$C = 6.2 \times 10^{-6}$
1	26 - 75	$0.5 - 1 \times 10^{-5}$
10	81 - 240	$2 - 5 \times 10^{-5}$
100	260 - 750	$0.5 - 1 \times 10^{-4}$

- For Transmitter Sizing, use 10nm filter and dim light: $N_{BG} \approx 10^{-4}$ noise electrons

INTERNAL NOISE

- Internal Noise Sources in CCDs (Dereniak & Crowe):
    - Input Noise
    - Transfer Efficiency Noise
    - Trapping Noise
    - Dark Current Noise
    - Clock Feedthrough Noise
    - Floating Diffusion Reset Noise
    - Amplifier Noise
    - Detector Uniformity Noise
    - Read Noise

- Typical Internal Noise values (all sources) range from 7 to 800 noise electrons per pixel.

  Fairchild CCD222 array has 60 noise electrons per pixel.

  IR Handbook gives typical value for state-of-the-art ccd cameras as 100 rms noise electrons.

$$\Rightarrow N_{IN} = 100 \text{ noise electrons}$$

TOTAL NOISE

- Total number of Noise Electrons is the root-sum-square addition of the contributions:

$$N_{Tot} = \left(\sum_i N_i^2\right)^{1\backslash 2}$$

- Total Noise Electrons (other than Signal Shot Noise):

$$N_{Tot} = (N_{BG}^2 + N_{IN}^2)^{1\backslash 2} \approx \begin{cases} 100 & \text{for no background contribution} \\ 100 & \text{for dim light in Collimator} \\ 410 & \text{for bright sunlight in Tower} \end{cases}$$

- Total number of Noise Electrons is standard deviation of a Poisson process. The mean of that process is:

$$N_N = (N_{Tot})^2 = N_{BG}^2 + N_{IN}^2 \approx \begin{cases} 10^4 & \text{for no background contribution} \\ 10^4 & \text{for dim light in Collimator} \\ 1.7 \times 10^5 & \text{for bright sunlight in Tower} \end{cases}$$

RADAR SIGNAL-TO-NOISE RATIO (SNR)

For any radar, the SNR is found by joint solution of:

Probability of False Alarm $$p_{fa} = \sum_{k=N_t}^{\infty} p_N(k)$$

Probability of Detection $$p_d = \sum_{k=N_t}^{\infty} p_{S+N}(k)$$

where $p_N(k)$ = probability of generating k electrons due to noise alone
$p_{S+N}(k)$ = probability of generating k electrons due to noise and signal
N_T = threshold level

114 Detection Statistics

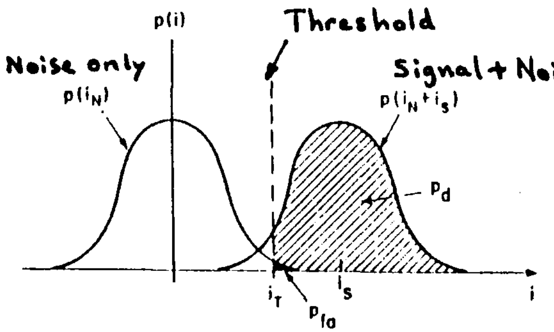


Fig. 10.1. Probability density for dc signal current in Gaussian noise

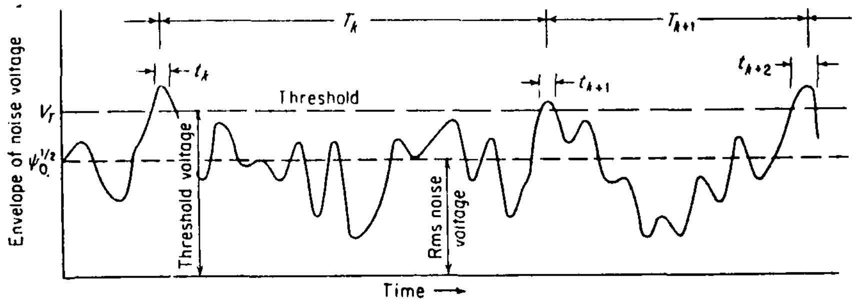


Figure 2.4 Envelope of receiver output illustrating false alarms due to noise.

DETECTION STATISTICS

• **Heterodyne Detection:**

Detected Power ∝ Received Power

Complex Gaussian Noise
Bandlimited
(Rayleigh Distribution):

$$p(i) = \frac{i}{\overline{i_N^2}} \exp\left(-\frac{i^2}{2\overline{i_N^2}}\right)$$

Signal + Noise
(Ricean Distribution):

$$p(i) = \frac{i}{\overline{i_N^2}} \exp\left(-\frac{(i^2 + i_S^2)}{2\overline{i_N^2}}\right) I_0\left(\frac{i\, i_S}{\overline{i_N^2}}\right)$$

where $I_0(\cdot)$ is the modified Bessel function of order 0

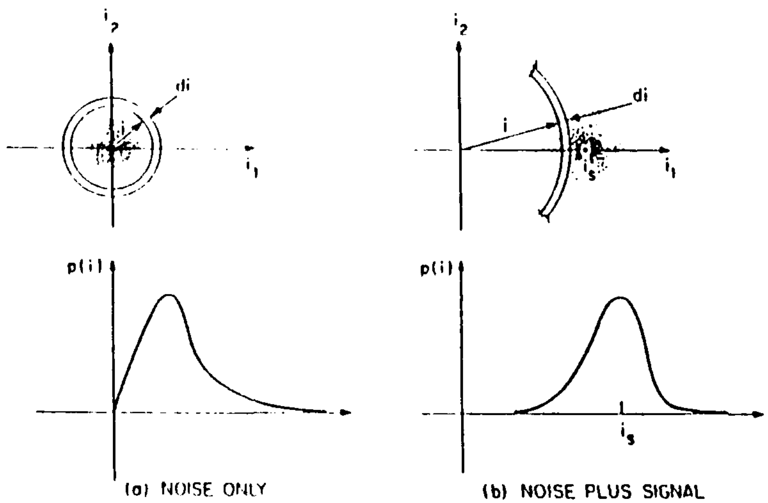


Fig. 10.3a and b. Probability density for envelope of sinusoidal signal in complex Gaussian noise

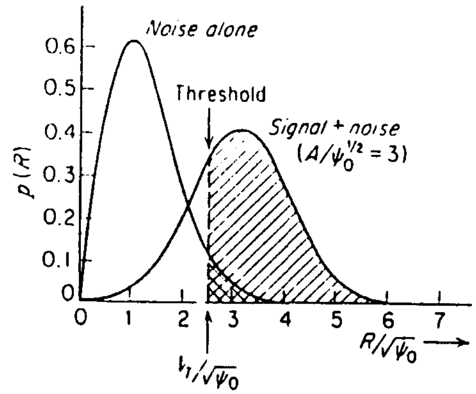


Figure 2.6 Probability-density function for noise alone and for signal-plus-noise, illustrating the process of threshold detection.

DETECTION STATISTICS (CONTINUED)

• **Direct Detection:**

Probability of Photoelectron Emission $\propto$ Optical Energy
(Detected Current $\propto$ Received Power)

Poisson Statistics valid for: $\frac{N_{Avg}}{\tau \Delta \nu} \ll 1$

1. Statistics in any disjoint time interval are independent.
2. As length of time interval $\Delta t \downarrow$, probability of >1 photelectron $\rightarrow$ negligib:y small and probability of exactly 1 photoelectron $\rightarrow \alpha \Delta t = N_{Avg}$ where α = average rate

Noise
(Poisson Distribution): $p_N(k) = \frac{(N_N)^k}{k!} \exp(-N_N)$

Signal
(Poisson Distribution): $p_S(k) = \frac{(N_S)^k}{k!} \exp(-N_S)$

Signal + Noise
(Poisson Distribution): $p_{S+N}(k) = \frac{(N_S + N_N)^k}{k!} \exp[-(N_S + N_N)]$

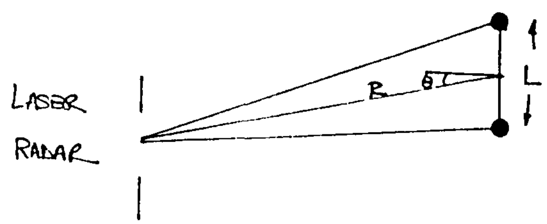
where N_N = average number of noise electrons generated
N_S = average number of signal electrons generated

TARGET STATISTICS: SPECKLE

- Speckle is variation of the target cross section that affects signal statistics ⇒ spatial fluctuation. Target return can be correlated or uncorrelated from pulse to pulse ⇒ temporal fluctuation.
- Energy reflected off target is redistributed into grainy pattern with peaks and nulls.
- Speckle is caused by surface roughness:

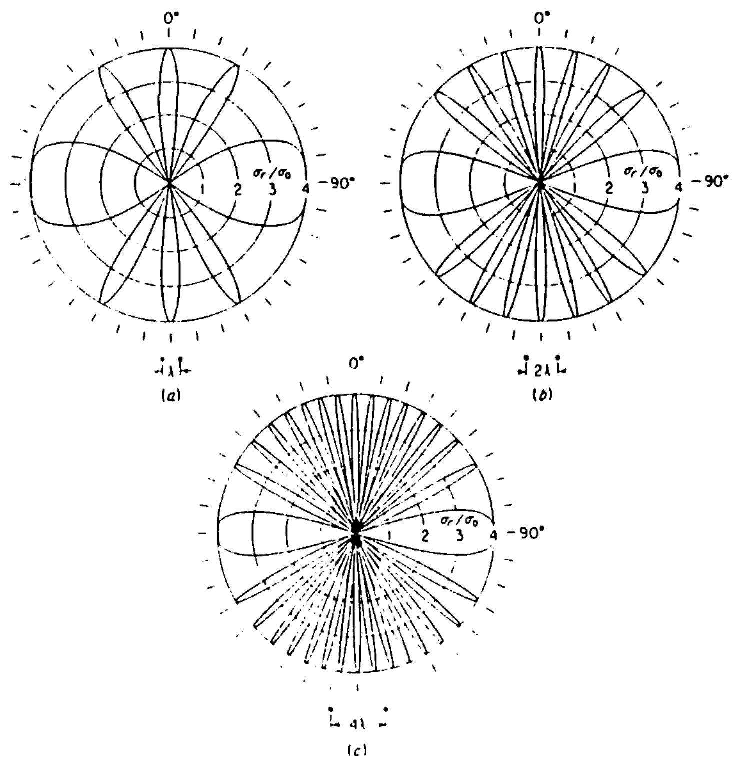
specular target ⇒ optically smooth	smaller features ⇒ larger speckle lobes
diffuse target ⇒ optically rough	larger features ⇒ smaller speckle lobes

TWO SPHERES



$$\frac{\sigma}{\sigma_0} = 2\left[1 + \cos\left(\frac{4\pi L}{\lambda}\sin\theta\right)\right]$$

σ_0 = CROSS SECTION OF ONE SPHERE



TARGET STATISTICS: SPECKLE (CONTINUED)

- Previous probability of detection calculations assumed a constant average signal from the target N_S. With speckle, the average signal is itself a random variable with probability density function $p(N_S)$.

Probability of False Alarm

$$p_{fa} = \sum_{k=N_t}^{\infty} p_N(k)$$

Probability of Detection

$$p_d = \int_0^{\infty} p_d(N_S)\,p(N_S)\,\mathrm{d}N_S$$

$$= \int_0^{\infty} \sum_{k=N_t}^{\infty} p_{S+N}(k)\,p(N_S)\,\mathrm{d}N_S$$

- Typical Speckle Statistics:

Negative Exponential Distribution:

$$p(N) = \frac{1}{N_{Avg}} e^{-\frac{N}{N_{Avg}}}$$

DIRECT DETECTION WITH SPECKLE

J.W. Goodman, "Some Effects of Target-Induced Scintillation on Optical Radar Performance," Proc IEEE, Vol 53(11), Nov 65.

- **Noise:**

Poisson Distribution
$$p_N(k) = \frac{(N_N)^k}{k!}\exp(-N_N)$$

- **Signal:**

Signal Spectral Energy Density w in each Correlation Cell (Speckle Lobe):

Negative Exponential Distribution
$$p(w) = \frac{1}{w_{Avg}}\exp\left(-\frac{w}{w_{Avg}}\right)$$

Total Received Energy $W = \int_{A_R} w\,\mathrm{d}A$:

Gamma Distribution
$$p(W) = \left(\frac{M}{W_{Avg}}\right)^M \frac{W^{M-1}\exp\left(-\frac{MW}{W_{Avg}}\right)}{\Gamma(M)}$$

where M = number of correlation cells

Probability of k Signal Photons $p_S(k) = \int_0^\infty p_S(k \mid W)p(W)\mathrm{d}W$ where $p_S(k \mid W) = \left(\frac{\eta W}{h\nu}\right)^k \exp\left(-\frac{\eta W}{h\nu}\right) / k!$:

Negative Binomial Distribution
$$p_S(k) = \frac{\Gamma(k+M)}{\Gamma(k+1)\Gamma(M)}\left(1+\frac{M}{N_S}\right)^{-k}\left(1+\frac{N_S}{M}\right)^{-M}$$

GAMMA FUNCTION

$$\Gamma(\alpha) = \int_0^\infty e^{-t} t^{\alpha-1} \mathrm{d}t \qquad (\alpha > 0)$$

$\Gamma(1) = 1$

$\Gamma(\alpha + 1) = \Gamma(\alpha)$

$\Gamma(k+1) = k! \qquad (k = 0, 1, \ldots)$

Generalized Factorial Function

For $\alpha < 0$:

$$\Gamma(\alpha) = \frac{\Gamma(\alpha+1)}{\alpha} = \frac{\Gamma(\alpha+2)}{\alpha(\alpha+1)} = \frac{\Gamma(\alpha+k+1)}{\alpha(\alpha+1)\cdots(\alpha+k)} \qquad (\alpha \neq 0, -1, -2, \ldots)$$

where k = smallest integer such that $\alpha + k + 1 > 0$.

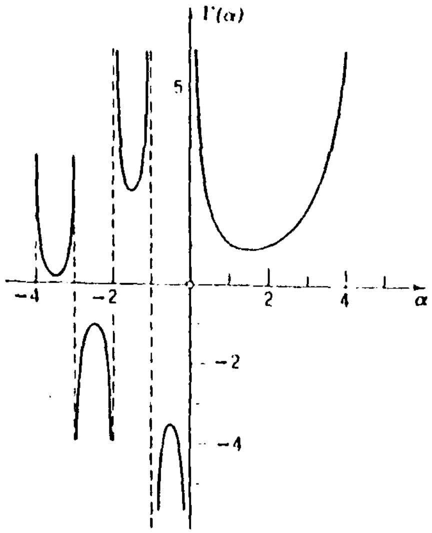


Fig. 405. Gamma function

Definition. Let X be a continuous random variable assuming only nonnegative values. We say that X has a *Gamma probability distribution* if its pdf is given by

$$f(x) = \frac{\alpha}{\Gamma(r)}(\alpha x)^{r-1}e^{-\alpha x}, \qquad x > 0$$
$$= 0, \qquad \text{elsewhere.} \qquad (9.16)$$

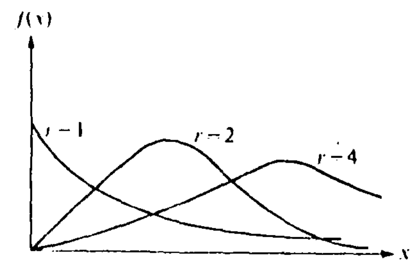


FIGURE 9.6

DIRECT DETECTION WITH SPECKLE (CONTINUED)

• **Signal + Noise**

Probability of k photoelectrons due to Negative Binomial Signal + Poisson Noise:

$$p_{S+N}(k) = \left(\frac{M}{M+N_S}\right)^M \frac{\exp(-N_N)}{(M-1)!} \sum_{j=0}^{k} \frac{(k+M-j-1)!}{j!(k-j)!} (N_N)^j \left(\frac{N_S}{M+N_S}\right)^{k-j}$$

• **Simplifications of Negative Binomial Signal:**

For M=1 (one speckle lobe received) $\Rightarrow$ diffuse, unresolved target:

Signal:
Bose-Einstein Distribution

$$p_S(k) = \frac{1}{1+N_S}\left(\frac{N_S}{1+N_S}\right)^k$$

Signal + Noise:

$$p_{S+N}(k) = \frac{\exp(-N_N)}{1+N_S} \sum_{j=0}^{k} \frac{(N_N)^j}{j!} \left(\frac{N_S}{1+N_S}\right)^{k-j}$$

For $\frac{N_S}{M} \ll 1 \Rightarrow$ diffuse, well-resolved target:

Signal:
Poisson Distribution

$$p_S(k) = \frac{(N_S)^k}{k!}\exp(-N_S)$$

Signal + Noise:
Poisson Distribution

$$p_{S+N}(k) = \frac{(N_{S+N})^k}{k!}\exp(-N_{S+N})$$

Resolved Target $\Rightarrow \downarrow$ range $\Rightarrow \uparrow$ number of Correlation Cells intercepted by receiver.

M=∞ $\Rightarrow$ identical to Specular.

SNR PLOTS

- Find threshold N_T, the minimum integer such that probability of noise exceeding N_T is less than desired p_{fa}:

Probability of False Alarm $$p_{fa} \geq \sum_{k=N_t}^{\infty} p_N(k) = \sum_{k=N_t}^{\infty} \frac{(N_N)^k}{k!} \exp(-N_N)$$

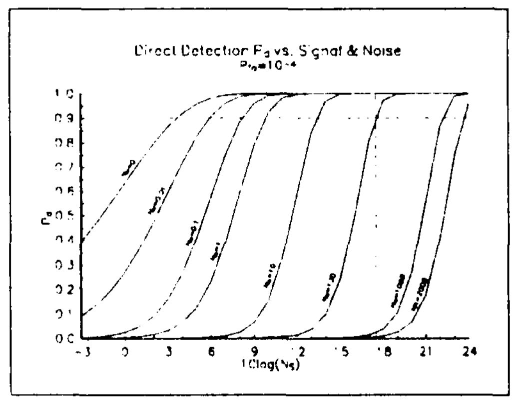
- Calculate probability that signal + noise exceeds N_T:

Probability of Detection $$p_d = \sum_{k=N_t}^{\infty} p_{S+N}(k) = \sum_{k=N_t}^{\infty} \frac{(N_N+N_S)^k}{k!} \exp[-(N_N+N_S)]$$

Neglect effects of speckle. Valid for specular & well-resolved diffuse targets.

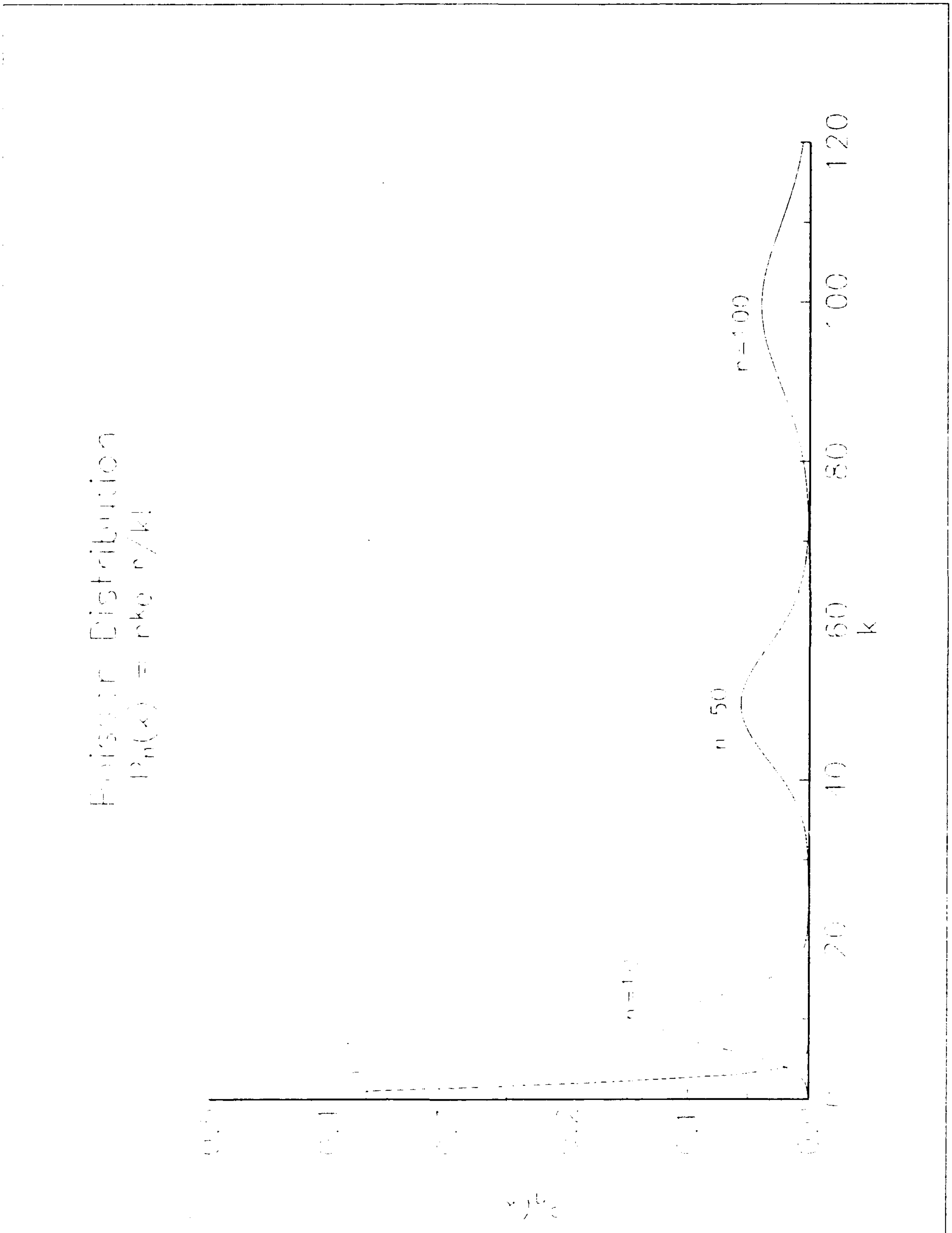
Note that N_N includes only noise other than signal shot noise

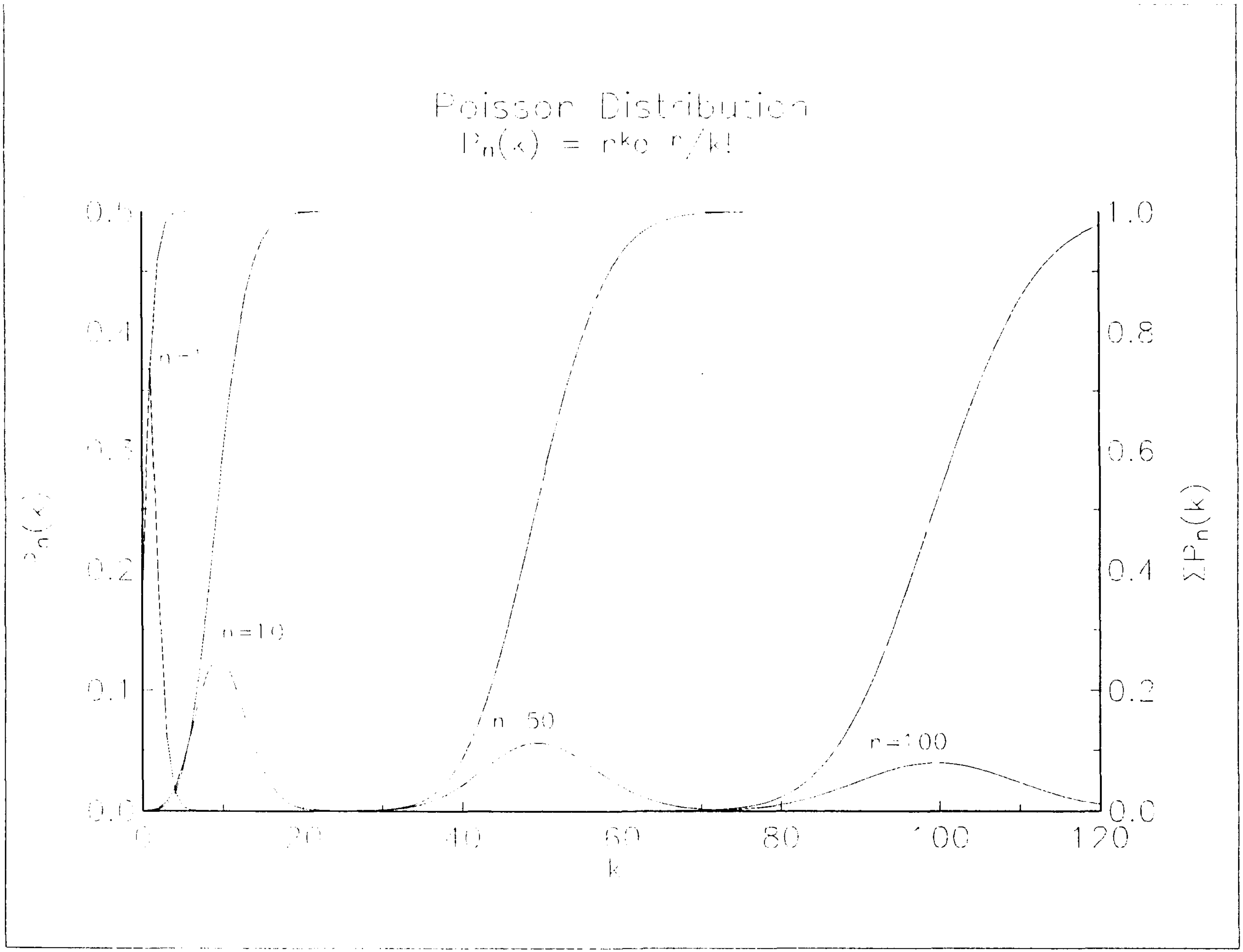
and that $p_{N+S}(k)$ takes into account signal shot noise.



- Plot p_d as a function of N_S for given p_{fa}.

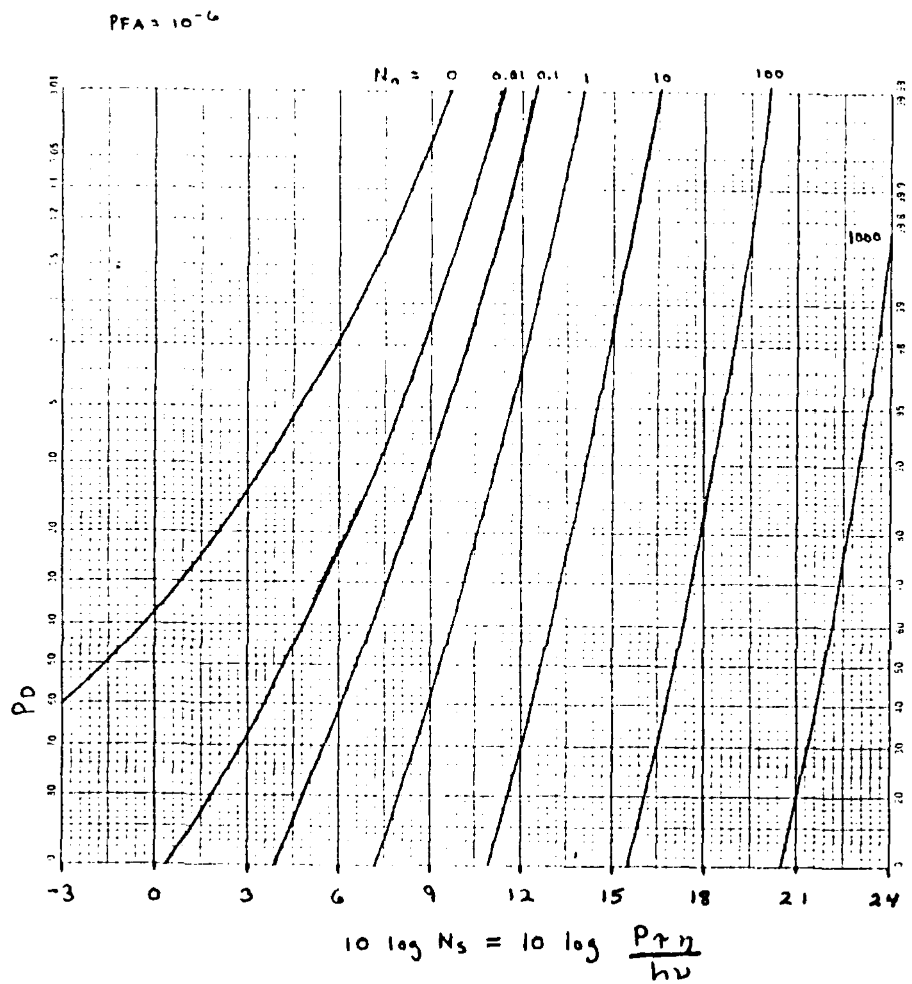
- For given N_N, read off $N_S \Rightarrow$ Minimum Detectable Signal.


Distribution
n 50
20
60
80
120
k


Poisson Distribution
$P_n(k) = n^k e^{-n}/k!$
$P_n(k)$
$\Sigma P_n(k)$
k
n=1
n=10
n=50
n=100
0.0
0.1
0.2
0.3
0.4
0.5
0.0
0.2
0.4
0.6
0.8
1.0
0
20
40
60
80
100
120

SNR PLOTS: Pd vs. SIGNAL & NOISE

Output of my computer program



Graph from Goodman's paper

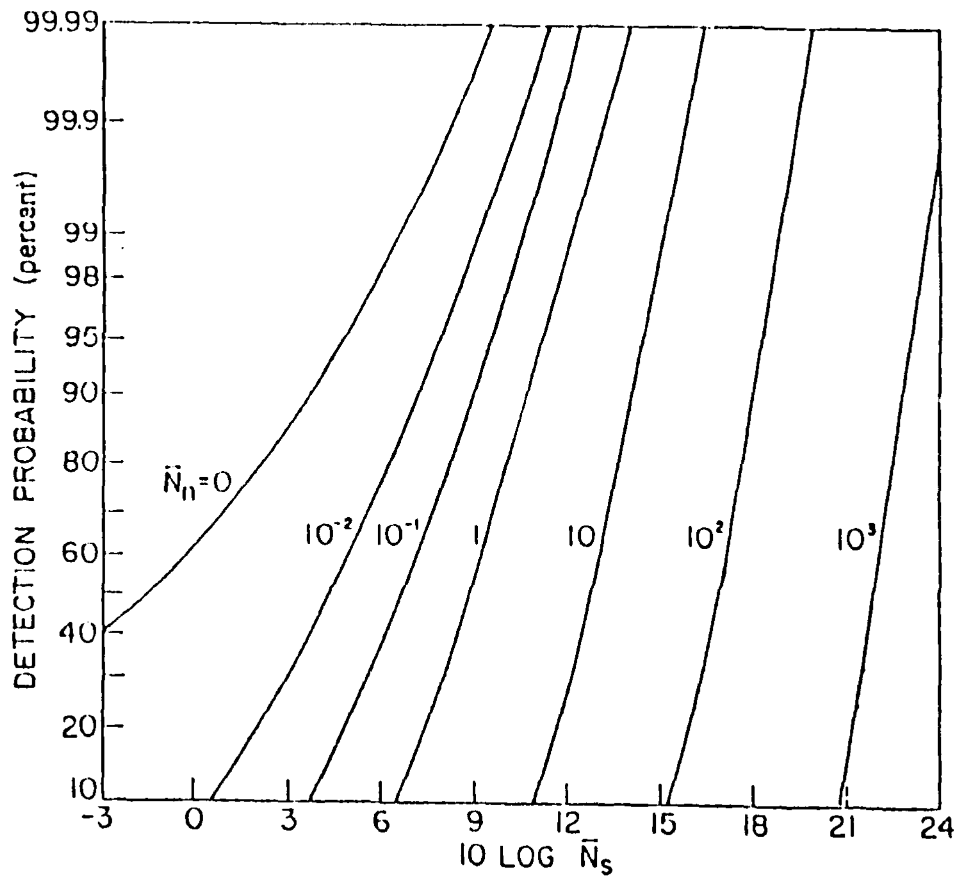
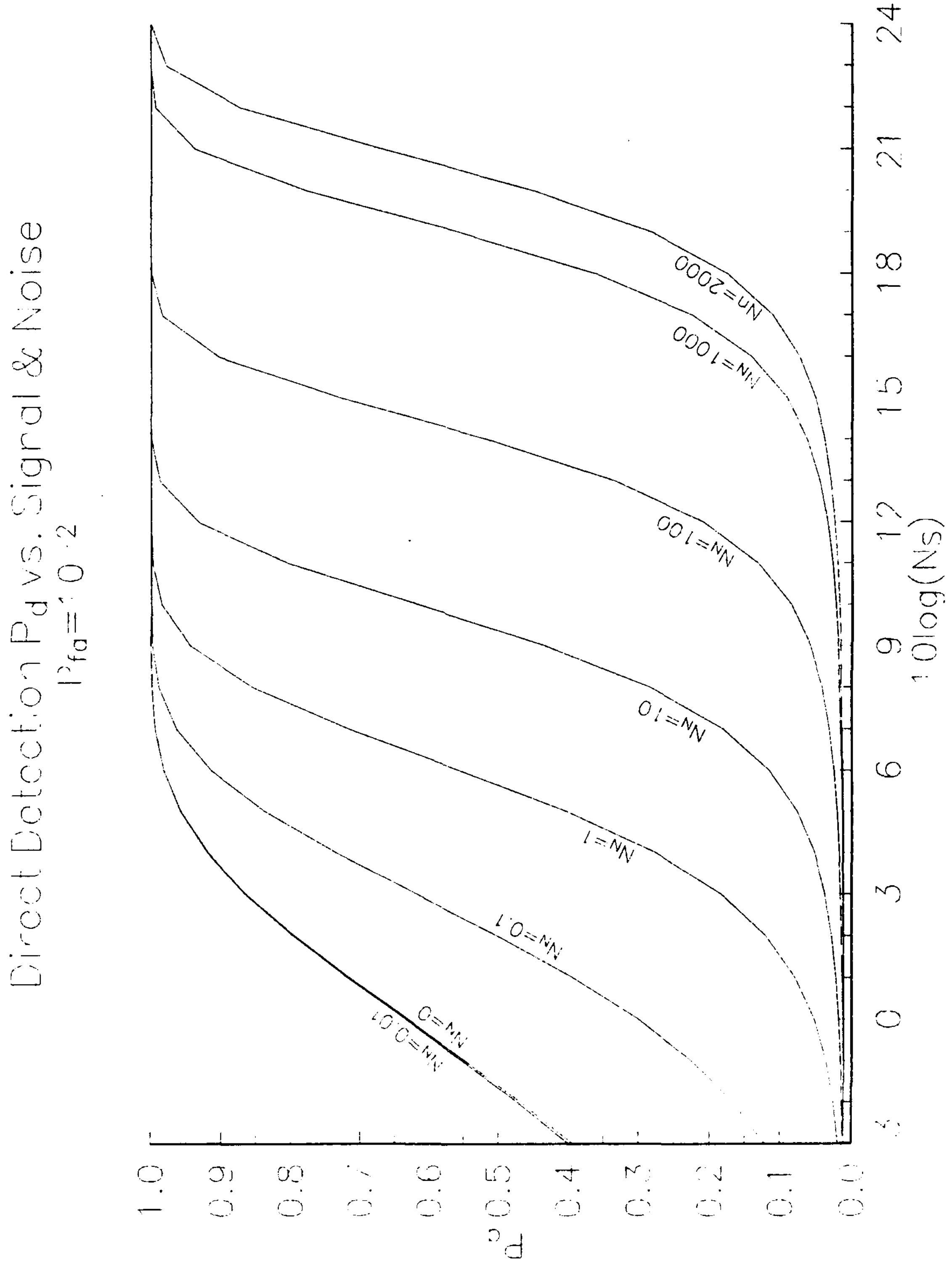
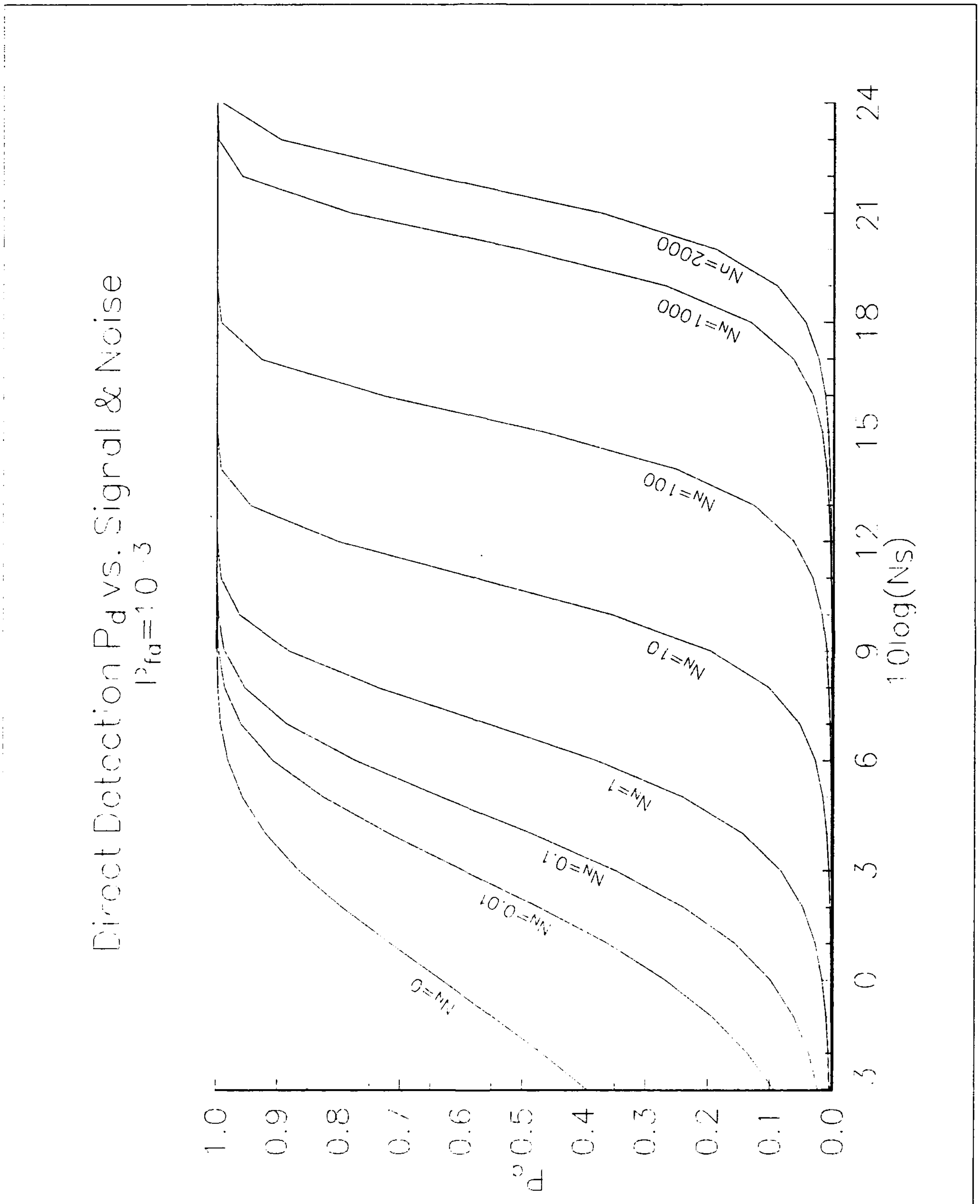
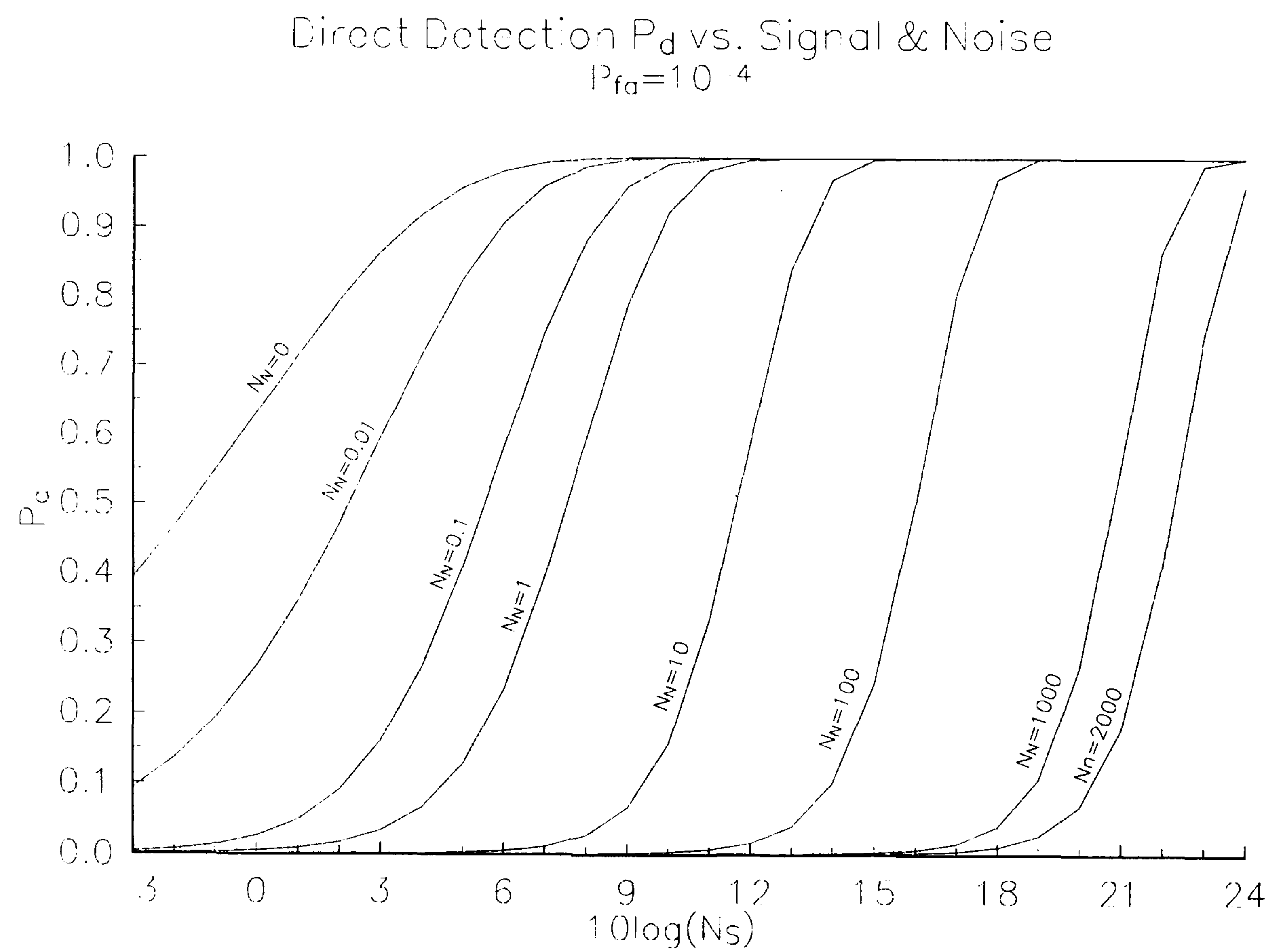


Fig. 2. Energy-detection radar performance—specular target $P_{FA} = 10^{-6}$.

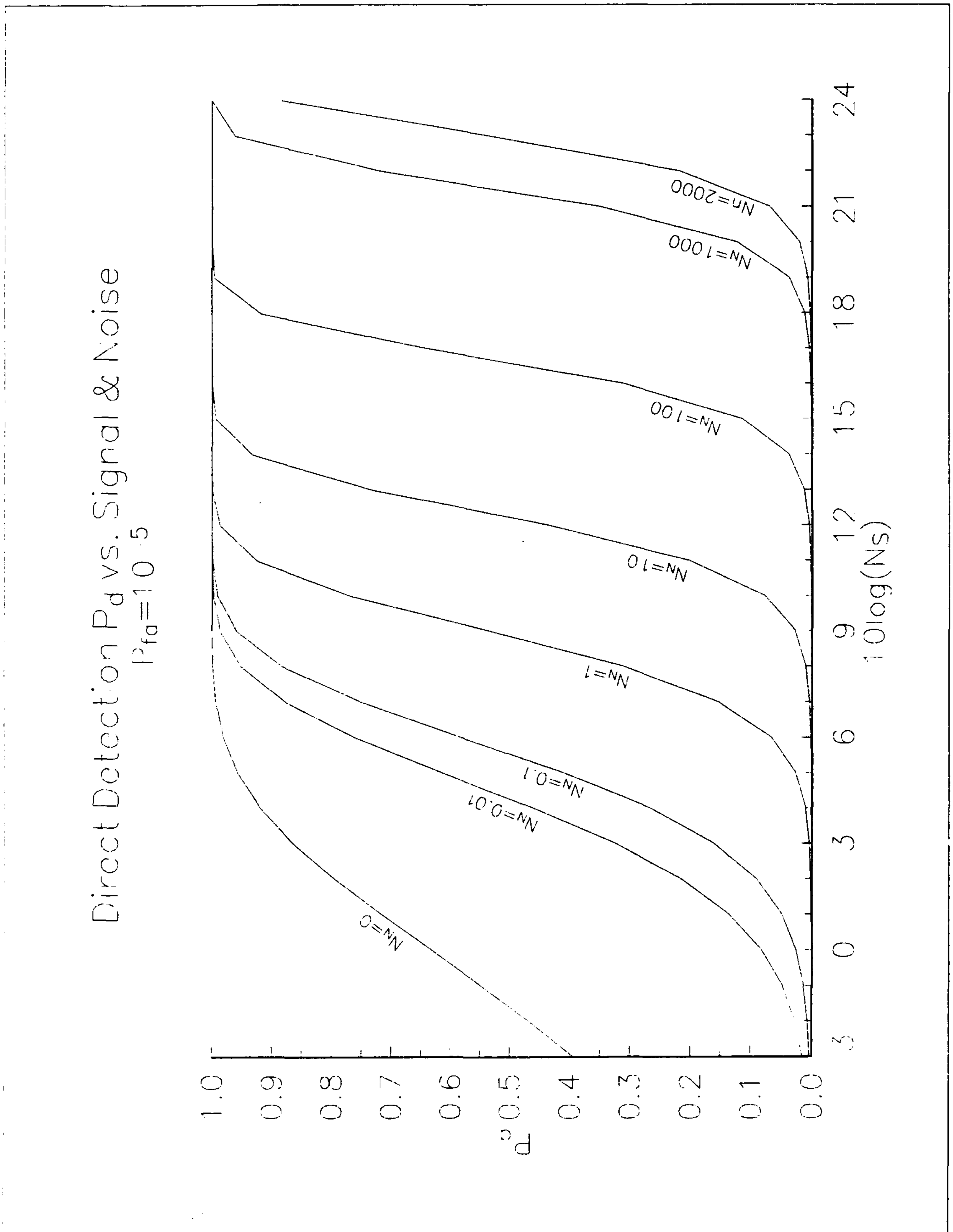


Direct Detection P_d vs. Signal & Noise
$P_{fa} = 10^{-2}$
P_d
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
-3
0
3
6
9
12
15
18
21
24
10log(Ns)
NN=0
NN=0.01
NN=0.1
NN=1
NN=10
NN=100
NN=1000
Nn=2000

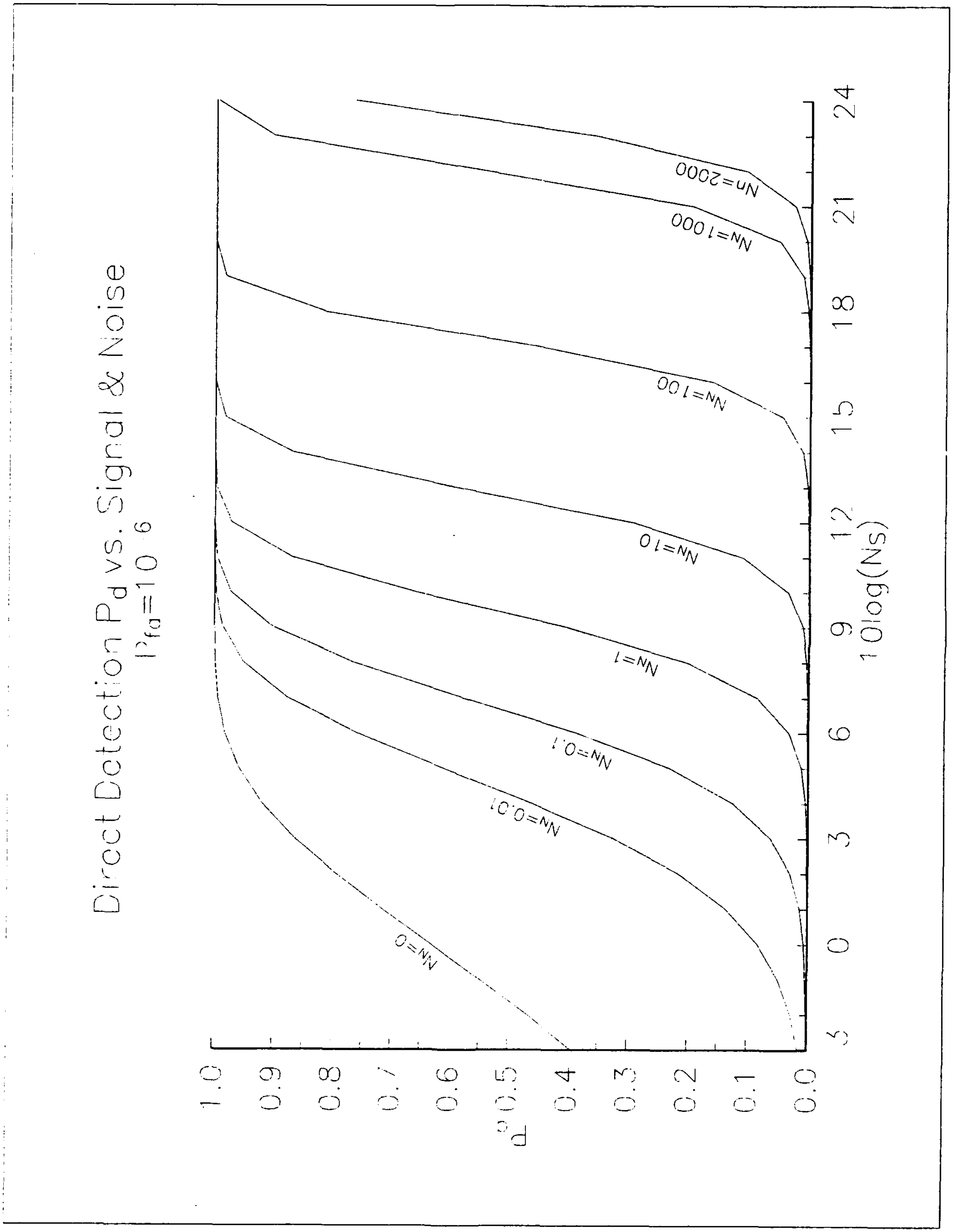

Direct Detection P_d vs. Signal & Noise
$P_{fa}=10^{-3}$
P_d
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
-3
0
3
6
9
12
15
18
21
24
10log(Ns)
Nn=0
Nn=0.01
Nn=0.1
Nn=1
Nn=10
Nn=100
Nn=1000
Nn=2000

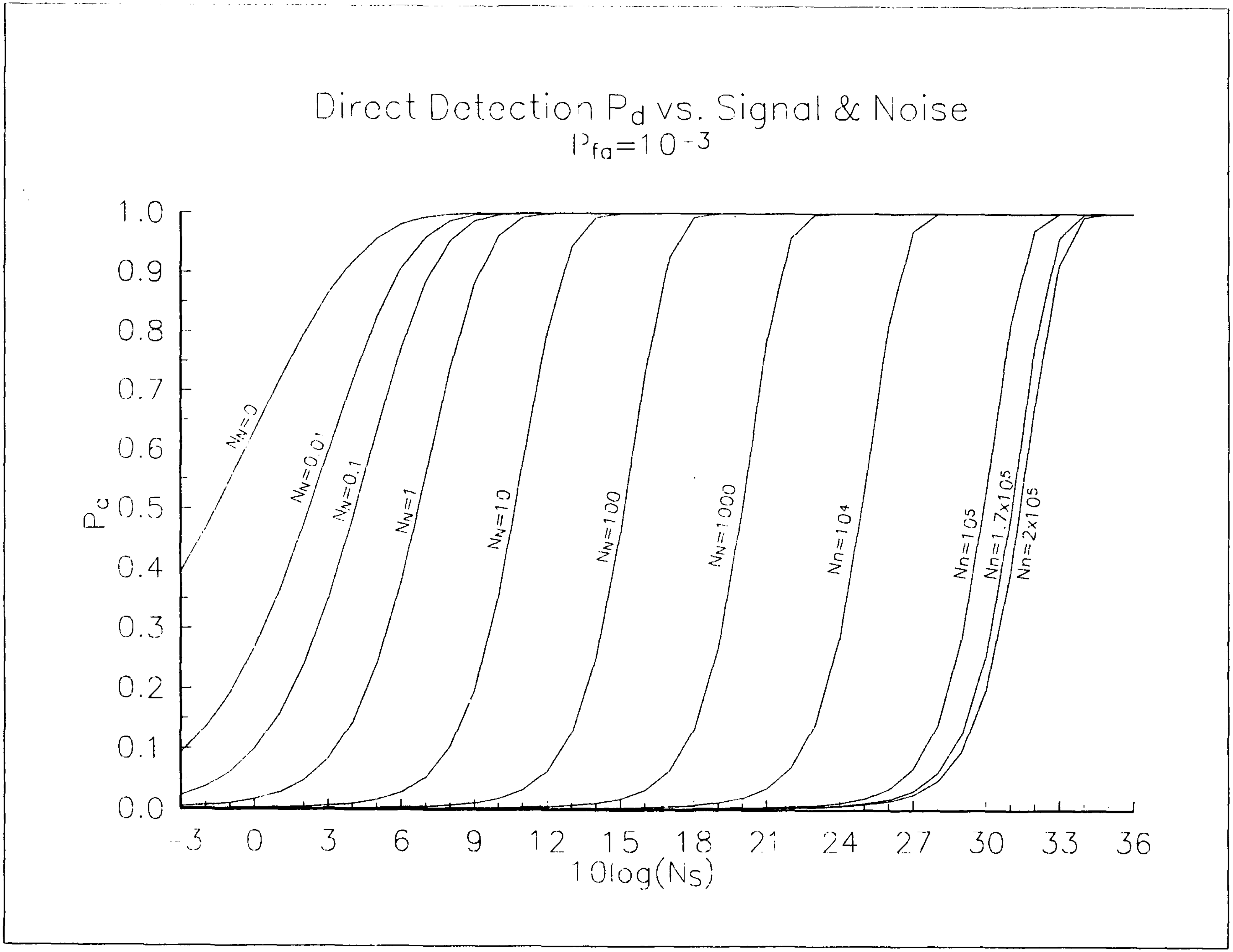


Direct Detection Pd vs. Signal & Noise
Pfa=10-4
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
Pc
-3
0
3
6
9
12
15
18
21
24
10log(Ns)
NN=0
NN=0.01
NN=0.1
NN=1
NN=10
NN=100
NN=1000
Nn=2000



Direct Detection Pd vs. Signal & Noise
Pfa=10 5
Pd
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
-3
0
3
6
9
12
15
18
21
24
10log(Ns)
Nn=0
Nn=0.01
Nn=0.1
Nn=1
Nn=10
Nn=100
Nn=1000
Nn=2000


Direct Detection P_d vs. Signal & Noise
$P_{fa}=10^{-6}$
P_d
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
-3
0
3
6
9
12
15
18
21
24
10log(Ns)
$N_N=0$
$N_N=0.01$
$N_N=0.1$
$N_N=1$
$N_N=10$
$N_N=100$
$N_N=1000$
$N_n=2000$


Direct Detection P_d vs. Signal & Noise
$P_{fa}=10^{-3}$
P_c
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
-3 0 3 6 9 12 15 18 21 24 27 30 33 36
10log(Ns)
$N_N=0$
$N_N=0.01$
$N_N=0.1$
$N_N=1$
$N_N=10$
$N_N=100$
$N_N=1000$
$Nn=10^4$
$Nn=10^5$
$Nn=1.7\times10^5$
$Nn=2\times10^5$

APPROXIMATIONS OF POISSON DISTRIBUTION

• **Bounds on P_{fa} & P_d in Closed Form:**

$$p_{fa} = \sum_{k=N_T}^{\infty} \frac{N_N^k e^{-N_N}}{k!} < e^{-N_N} \frac{N_N^{N_T}}{N_T!} \frac{1}{1-\left(\frac{N_N}{N_T+1}\right)}$$

$$p_d = 1 - P_{miss} = 1 - \sum_{k=0}^{N_T-1} \frac{N_{S+N}^k e^{-N_{S+N}}}{k!} > 1 - \left[e^{-N_{S+N}} \frac{N_{S+N}^{N_T-1}}{(N_T-1)!} \frac{1-\left(\frac{T-1}{N_{S+N}}\right)^{N_T}}{1-\left(\frac{T-1}{N_{S+N}}\right)} \right]$$

• **Gaussian Approximation for large N (N>100):**

Sum of N independent Random Variables $\rightarrow$ Gaussian Distribution as $N \rightarrow \infty$.

Mean $\rightarrow \Sigma$ Means Variance $\rightarrow \Sigma$ Variances

$$p_N(k) = \frac{N^k e^{-N}}{k!} \underset{N\rightarrow\infty}{\rightarrow} p_N(k) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(k-\mu)^2}{2\sigma^2}} \quad \text{where } \mu = \sigma^2 = N$$

<u>For $P_{fa} = 10^{-3}$:</u>

$N_S = 3.08\sqrt{N_N} + 1.29\sqrt{N_{S+N}}$

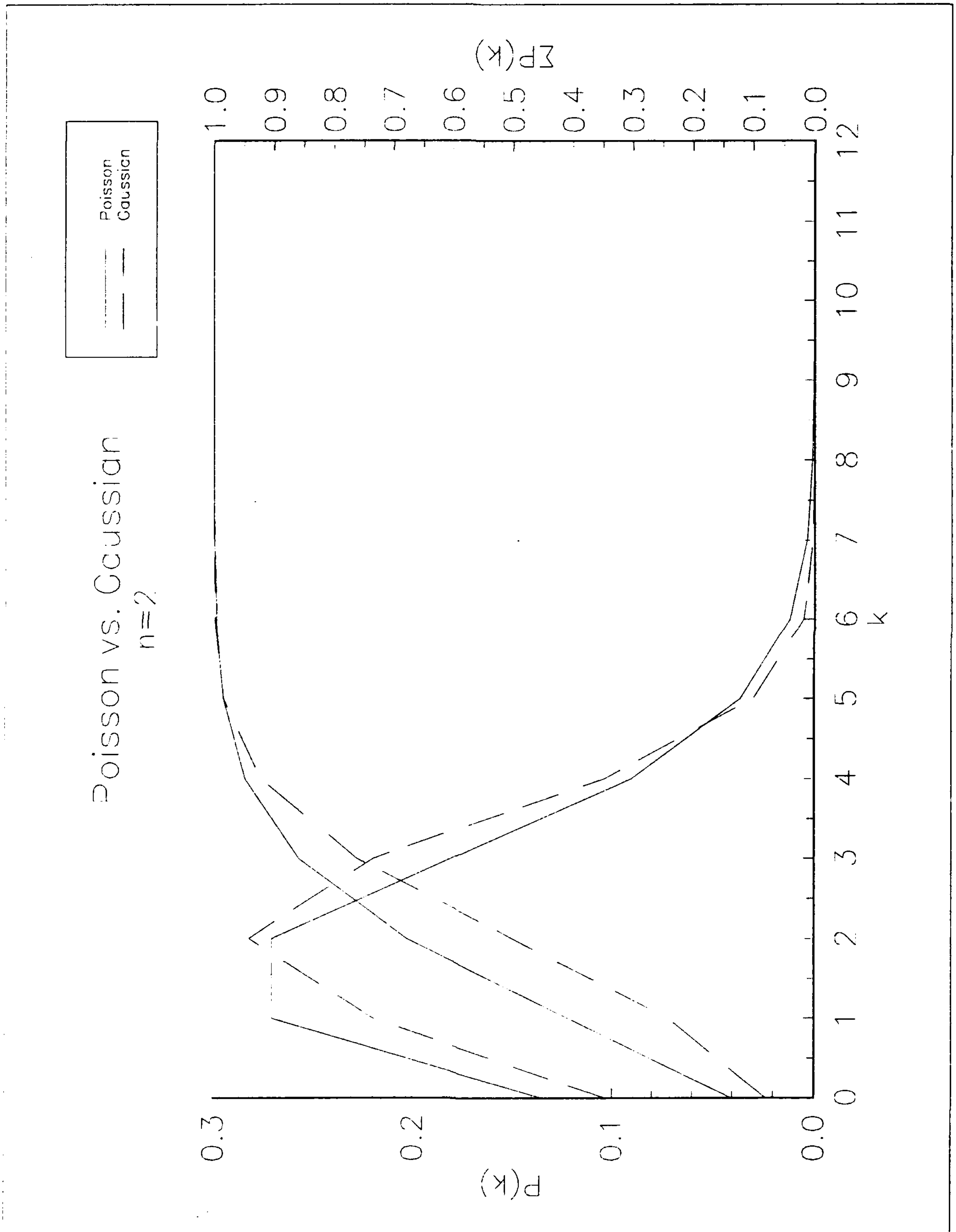
For large N_N: $N_S \approx 4.37\sqrt{N_N}$

<u>For $P_{fa} = 10^{-4}$:</u>

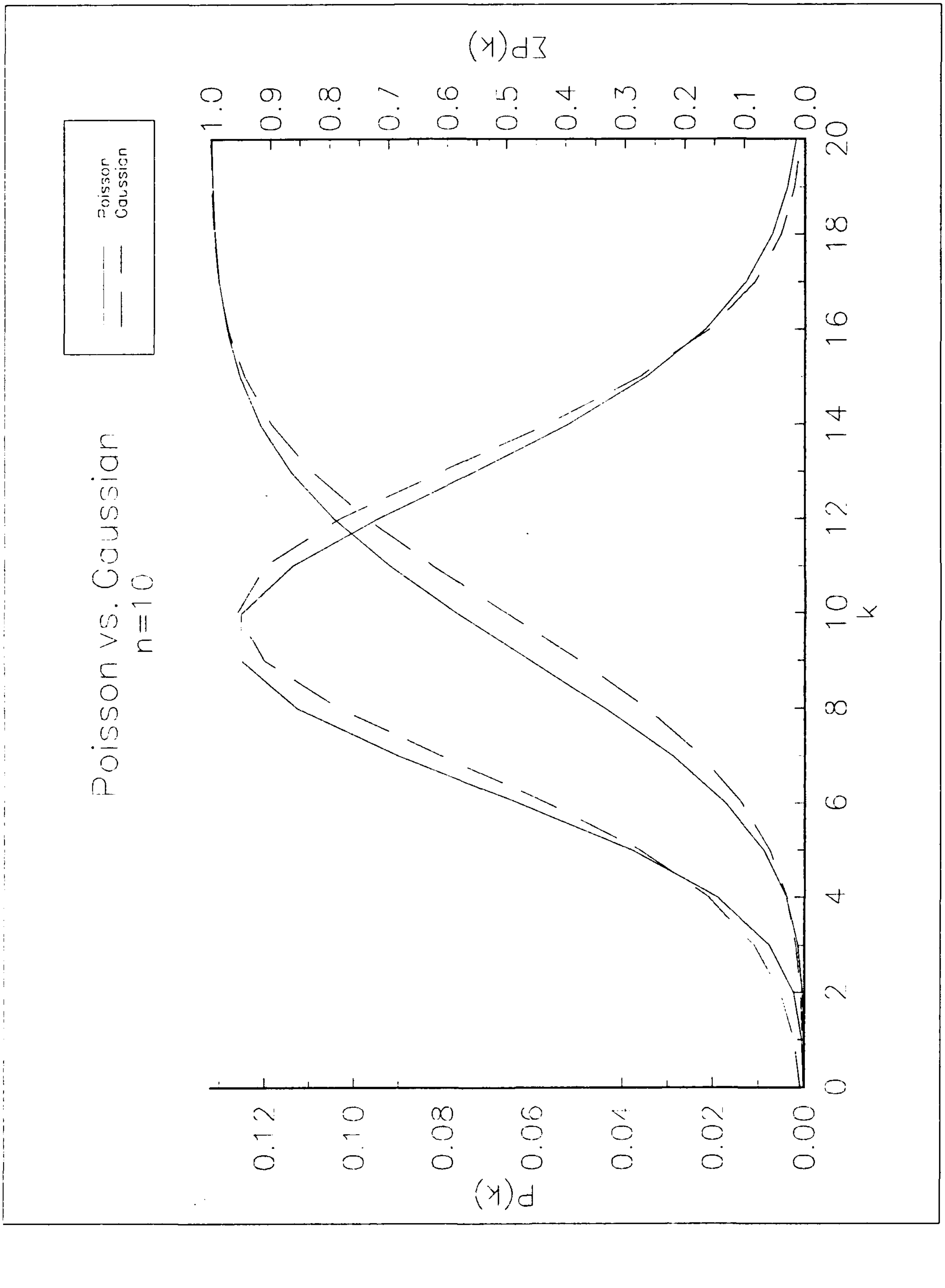
$N_S = 3.75\sqrt{N_N} + 1.29\sqrt{N_{S+N}}$

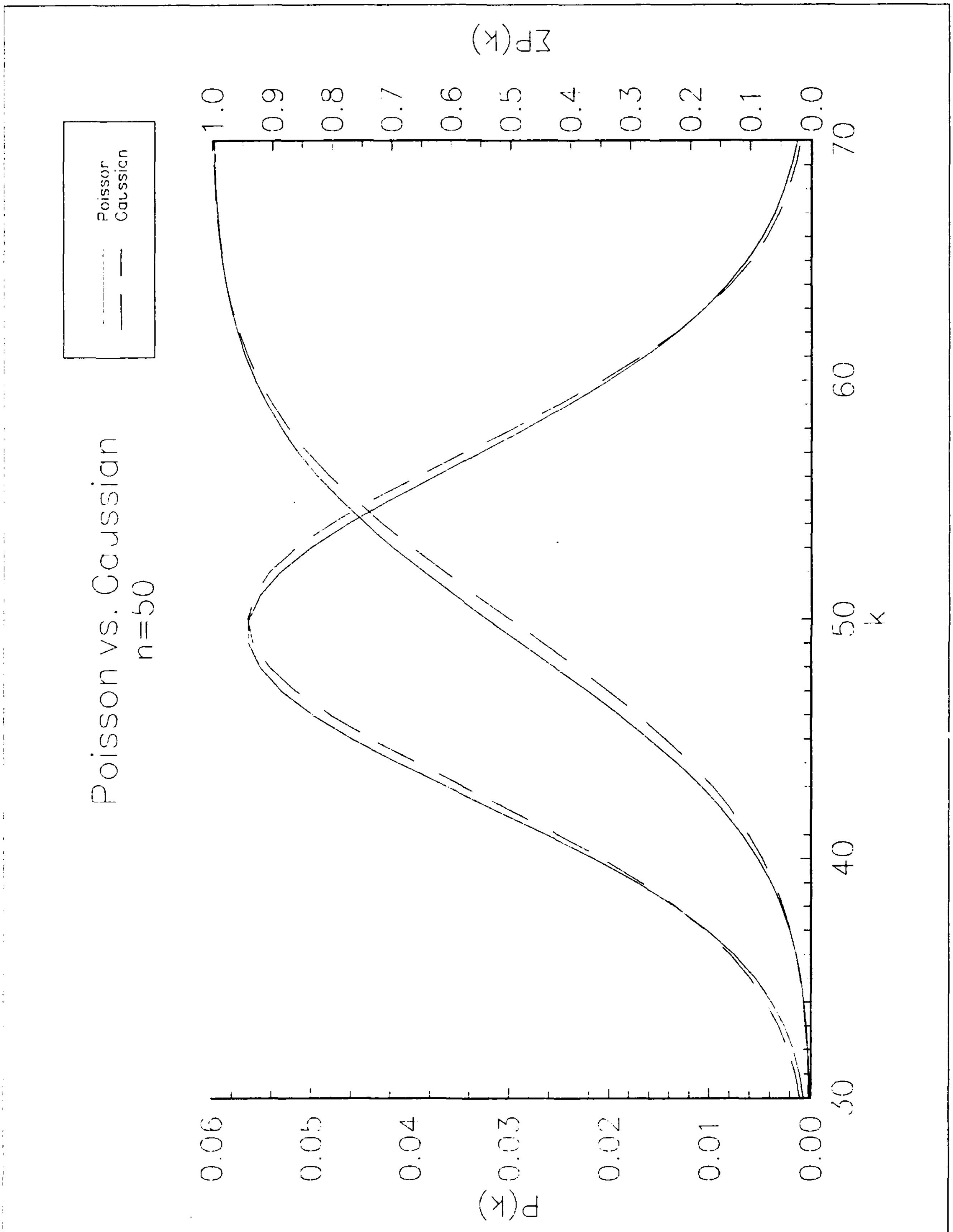
For large N_N: $N_S \approx 5.04\sqrt{N_N}$

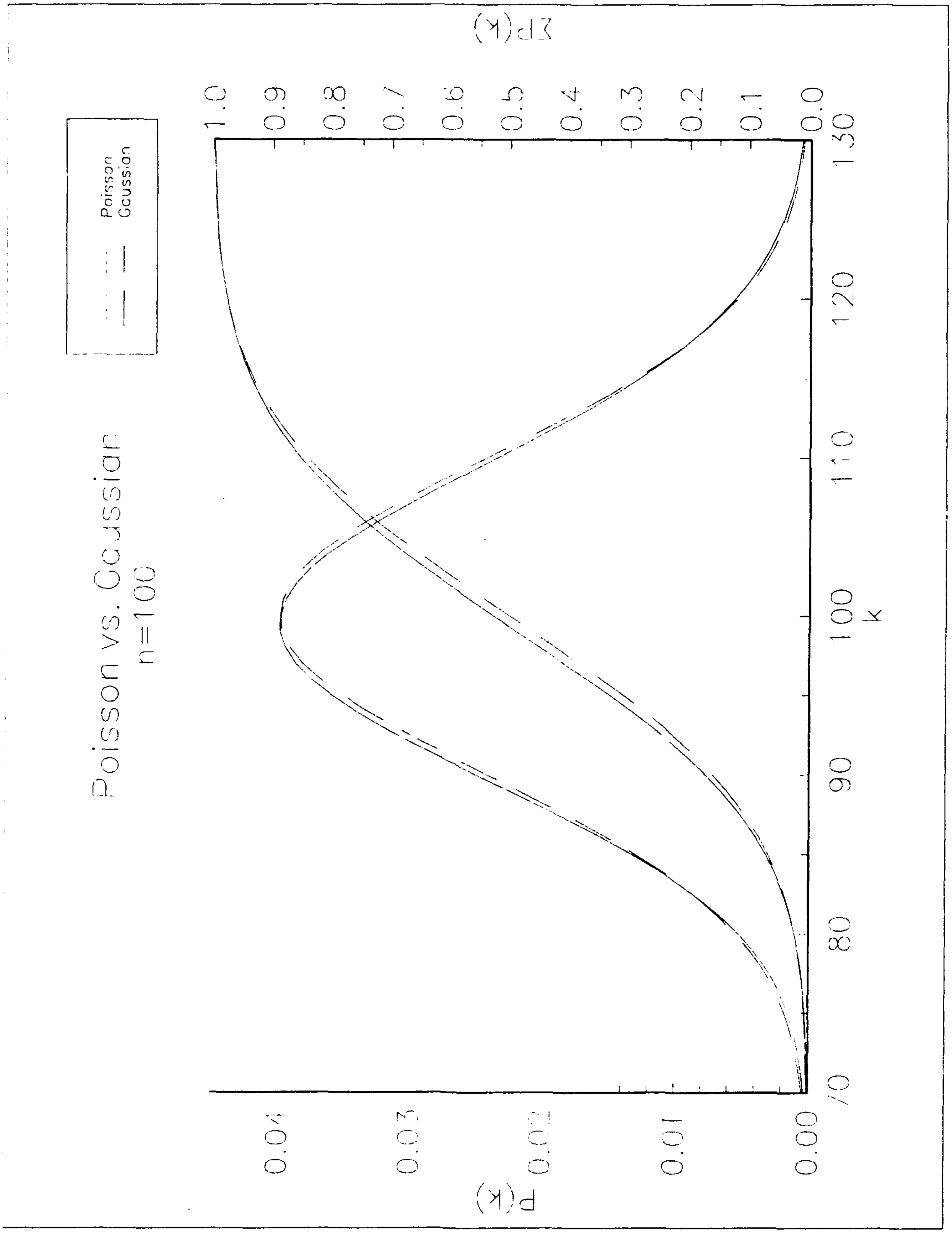
Note $N_{S+N} = N_S + N_N$

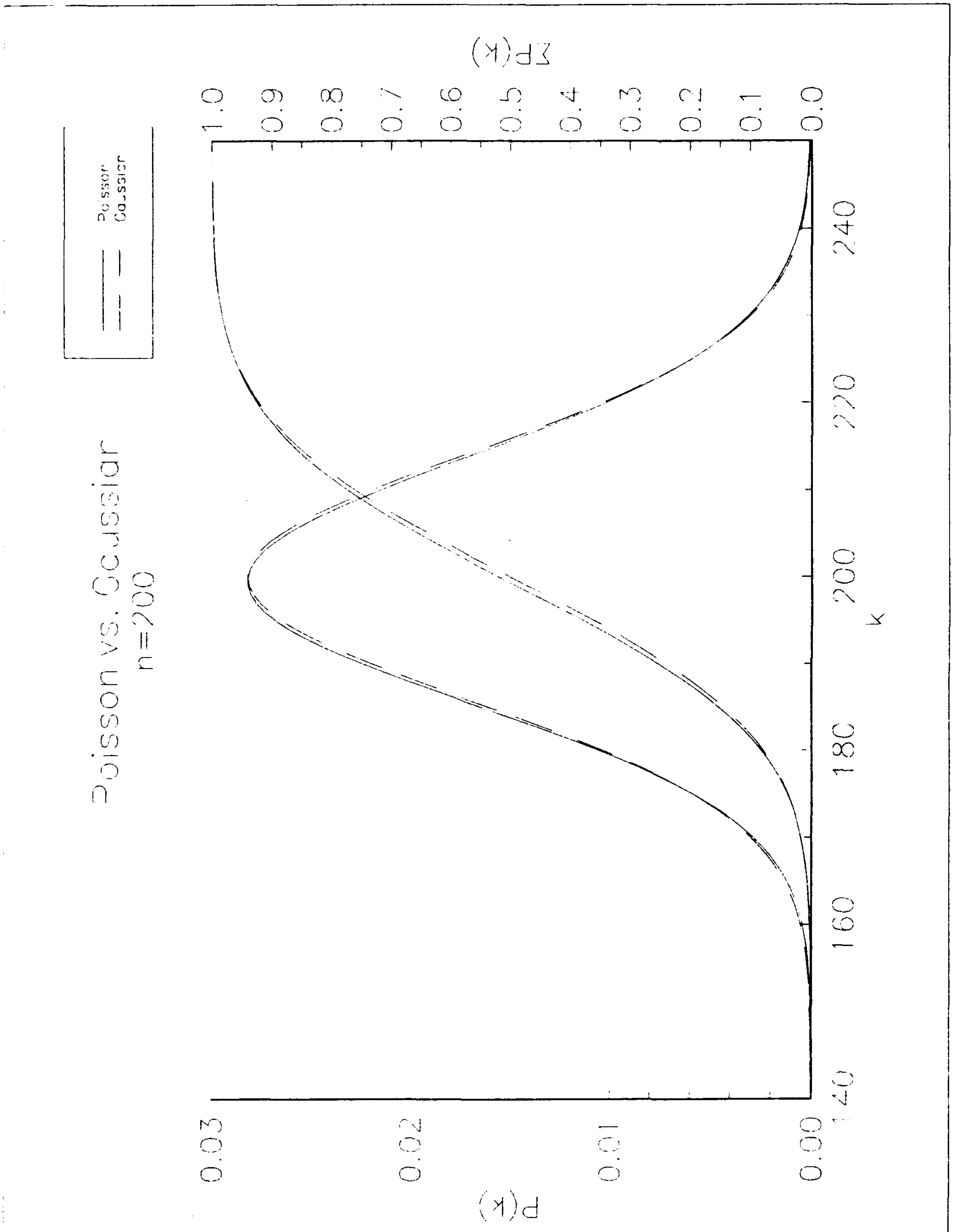


Poisson vs. Gaussian
n=2
Poisson
Gaussian
P(k)
ΣP(k)
k
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
0
1
2
3
4
5
6
7
8
9
10
11
12


Poisson vs. Gaussian
n=10
Poisson
Gaussian
P(k)
ΣP(k)
k
0.00
0.02
0.04
0.06
0.08
0.10
0.12
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
0
2
4
6
8
10
12
14
16
18
20


Poisson vs. Gaussian
n=50
Poisson
Gaussian
P(k)
ΣP(k)
k
0.00
0.01
0.02
0.03
0.04
0.05
0.06
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
30
40
50
60
70


Poisson vs. Gaussian
n=100
Poisson
Gaussian
P(k)
ΣP(k)
k
0.00
0.01
0.02
0.03
0.04
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
70
80
90
100
110
120
130


Poisson vs. Gaussian
n=200
Poisson
Gaussian
P(k)
ΣP(k)
k
0.00
0.01
0.02
0.03
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
160
180
200
220
240

VALUES OF P_d and P_{fa}

- **Probability of False Alarm, p_{fa}:**

$p_{fa} = \frac{\tau}{t_{fa}}$ where t_{fa} = time between false alarms

For imaging sensor, p_{fa} = Fraction of False Detection Pixels per frame.

Assume ergodic process so that temporal average = ensemble average, valid for Poisson statistics.

For initial LIMARS experiment, prefer image relatively free of false detections.

Use Fraction of False Detection Pixels <0.1%

$\Rightarrow$ 70 false detection pixels in frame of 256×256

$\Rightarrow p_{fa} = 10^{-3}$

- **Probability of Detection, p_d:**

$p_d = 1 - p_{DO}$ where p_{DO} = fraction of drop-out pixels

For initial LIMARS experiment, prefer relatively few drop-outs.

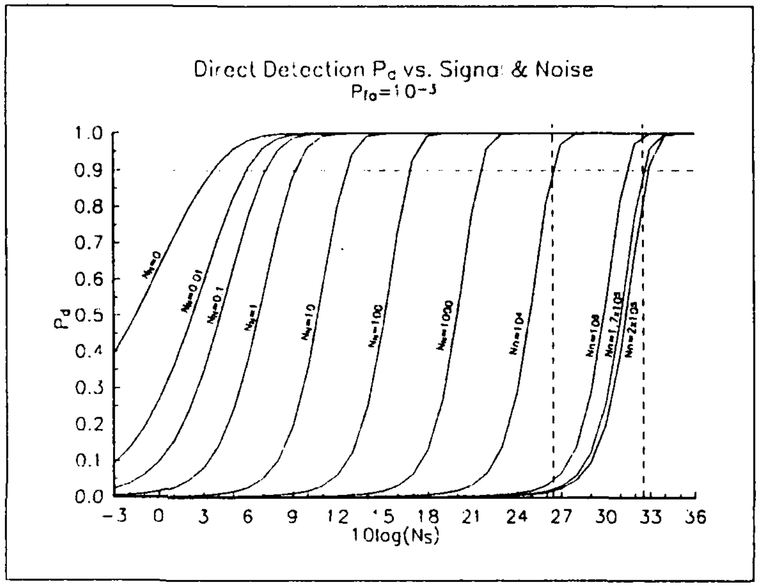
However SNR climbs rapidly for p_d>90%.

Use $p_d = 0.90$

$\Rightarrow$ 10% of target image pixels will be missing.

MINIMUM DETECTABLE SIGNAL FOR LIMARS

Assume $p_{fa} = 10^{-3}$
and $p_d = 0.90$:



N_N	N_S	E_S (J)*, $\eta = 1$	E_S (J)*, $\eta = 0.02$
10^4	440	8.3×10^{-17}	4.1×10^{-15}
1.7×10^5	1800	3.4×10^{-16}	1.7×10^{-14}

* Minimum Detectable Energy per pixel at detector for single pulse
$E_S = N_S hc/\lambda\eta$ where η=detector quantum efficiency.
(Recall that N_N was calculated with $\eta = 0.02$.)

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